

Factoring Perfect Squares and a Difference of Squares



Level 1 - Factor simple perfect squares and simple difference of squares

Level 2 - Factor more complex expressions where a GCF may be present

Level 3 - Factor complex expressions which may include two or more variables

Perfect Squares and Difference of Squares are special cases of quadratic trinomials that allow for quicker methods of factoring.

Difference of Squares

A difference of squares occurs when you subtract two perfect squares. It always factors as follows:

$$a^2 - b^2 = (a - b)(a + b)$$

Perfect Squares

A perfect square occurs when the quadratic is in the form:

$$a^2 + 2ab + b^2 \quad \text{or} \quad a^2 - 2ab + b^2$$

which factors to:

$$(a + b)^2 \quad \text{or} \quad (a - b)^2$$

Remember: Always factor out a GCF first!

| Example #1 | Example #2 | Example #3 | Example #4 |
|---|---|---|---|
| $x^2 - 9$ | $3x^2 - 48$ | $x^2 + 4x + 4$ | $2x^2 - 12x + 18$ |
| <p>$= x^2 + 0x - 9$</p> <p>$c = -9$ $b = 0$</p> <p>two numbers that multiply to give -9 and add to give 0 are 3 and -3.</p> <p>So,</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;">$(x + 3)(x - 3)$</div> | <p>factor out a GCF:</p> <p>$3(x^2 - 16)$</p> <p>factor the difference of squares</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;">$3(x + 4)(x - 4)$</div> | <p>check for a perfect square:</p> <p>$a = 1x^2 \checkmark$ $b = (2)^2 \checkmark$</p> <p>middle term check $= 2(a)(b) = 2(1x)(2)$ $= 4 \checkmark$</p> <p>the middle term is positive so,</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;">$(x + 2)^2$</div> | <p>factor out a GCF:</p> <p>$2(x^2 - 6x + 9)$</p> <p>check for a perfect square in the brackets:</p> <p>$a = 1x^2 \checkmark$ $b = (3)^2 \checkmark$</p> <p>middle term check $= 2(a)(b) = 2(1x)(3)$ $= 6 \checkmark$</p> <p>the middle term is negative so,</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;">$2(x - 3)^2$</div> |

Remember:

- For difference of squares, always check that both terms are perfect squares and that it's subtraction (never addition).
- For perfect square trinomials, test if the **middle term** equals **2ab**. If not, it's not a perfect square.