

Introduction to Imaginary Numbers



Level 1 - Simplify radicals with a negative radicand.

Level 2 - Simplify expressions involving imaginary numbers

Level 3 - Simplify expressions by rationalizing the denominator

When we take the square root of a negative number, we run into a problem!
No real number squared gives a negative.

To fix this, mathematicians created the imaginary unit:

Imaginary Unit
$i = \sqrt{-1}$

So $i^2 = -1$. We can perform operations and simplify using this imaginary unit (i).

Example #1	Example #2	Example #3	Example #4
$\sqrt{-16}$	$\sqrt{-4} - 2i$	$(\sqrt{-2} + i)(\sqrt{-2} - i)$	$\frac{10}{\sqrt{-20}}$
$\sqrt{16} \cdot \sqrt{-1}$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 0 auto;">4i</div>	$\sqrt{4} \cdot \sqrt{-1} - 2i$ $2i - 2i$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 0 auto;">0</div>	<p>First: $\sqrt{-2} \cdot \sqrt{-2} = (\sqrt{-2})^2 = -2$</p> <p>Outside: $\sqrt{-2} \cdot (-i) = -i\sqrt{-2}$</p> <p>Inside: $i \cdot \sqrt{-2} = i\sqrt{-2}$</p> <p>Last: $i \cdot (-i) = -i^2 = -(\sqrt{-1})^2 = +1$</p> $-2 - i\sqrt{-2} + i\sqrt{-2} + 1$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 0 auto;">-1</div>	$\frac{10}{\sqrt{20} \cdot \sqrt{-1}}$ $\frac{10}{2\sqrt{5} \cdot i}$ $\frac{5}{i\sqrt{5}}$ $\frac{5}{i\sqrt{5}} \cdot \frac{i\sqrt{5}}{i\sqrt{5}}$ $\frac{5i\sqrt{5}}{-5}$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 0 auto;">$-i\sqrt{5}$</div>

Remember:

- Always replace i^2 with **-1**. This keeps expressions simplified.
- Treat i just like a variable when multiplying or dividing, then simplify using $i^2 = -1$.
- To simplify fractions with i in the denominator, multiply top and bottom by i . This clears out the imaginary part from the denominator.