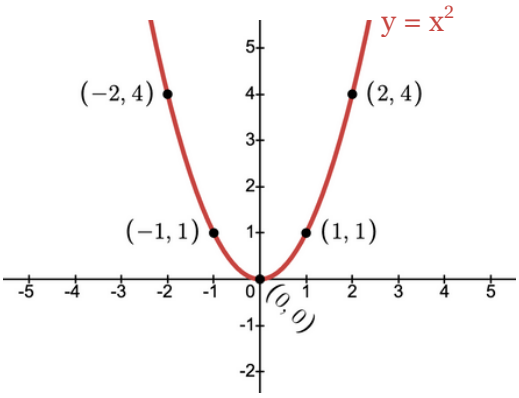
Graphing Quadratic Equations

Level 1 - Graph quadratic equations given in vertex form
Level 2 - Create a quadratic equation from a graph
Level 3 - Convert from standard to vertex form and graph the quadratic equation

Quadratic equations create graphs called parabolas. Learning how to graph these equations helps you visualize how the equation behaves and understand key features like the vertex, axis of symmetry, and direction of opening.



The parabola to the right is the graph of the quadratic parent function, $y = x^2$. All other quadratic graphs are manipulations of this one.

We can sketch a graph if the equation is given to us in vertex form:

$$y = a(x - h)^2 + k$$

How it changes the parent function...

- moves the vertex from (0, 0) to (h, k)
- stretches the graph vertically by a factor of "a"
- "+a" has the graph open up, "-a" has the graph open down

The graph of our parent function shows that the integer points next to the vertex follow a pattern.
First point: over 1 and up 1 from the vertex
Second point: over 2 and up 4 from the vertex

We use this to determine where these points will be when we stretch our graph vertically with an "a" value that is not 1.

Example #1

$y = 2x^2$

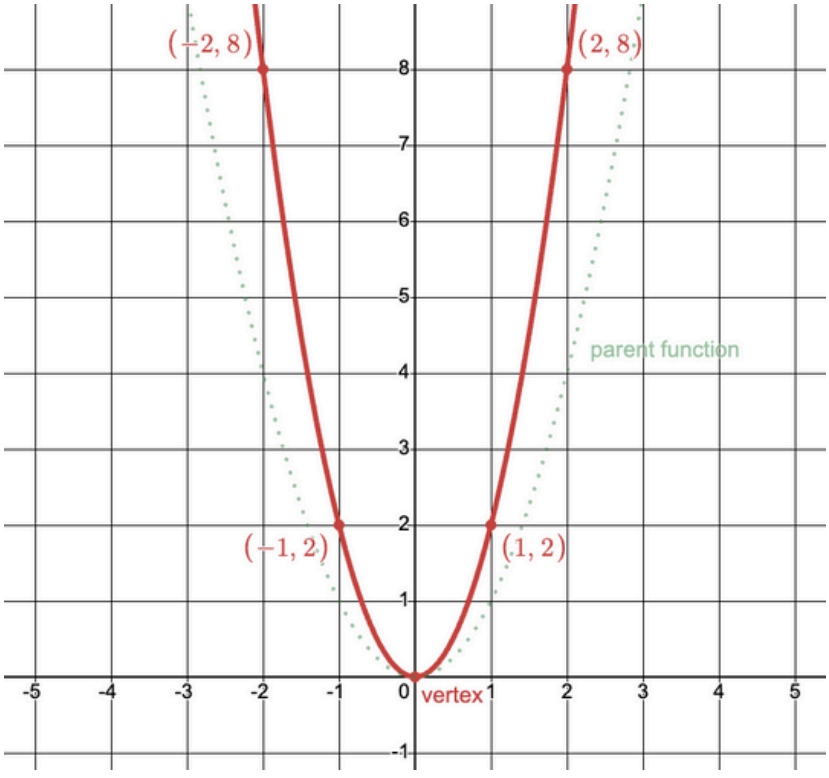
vertex form :
$y = 2(x - 0)^2 + 0$

vertex :
$(h, k) \rightarrow (0, 0)$

shape :
a = 2 so the y - values of the parent function get multiplied by 2

Rather than the closest point being over 1 and up 1, it will be over 1 and **up 2.**

Likewise the next point will be over 2 and **up 8.**



Example #2

$y = -x^2 + 2$

vertex form :
$y = -(x - 0)^2 + 2$

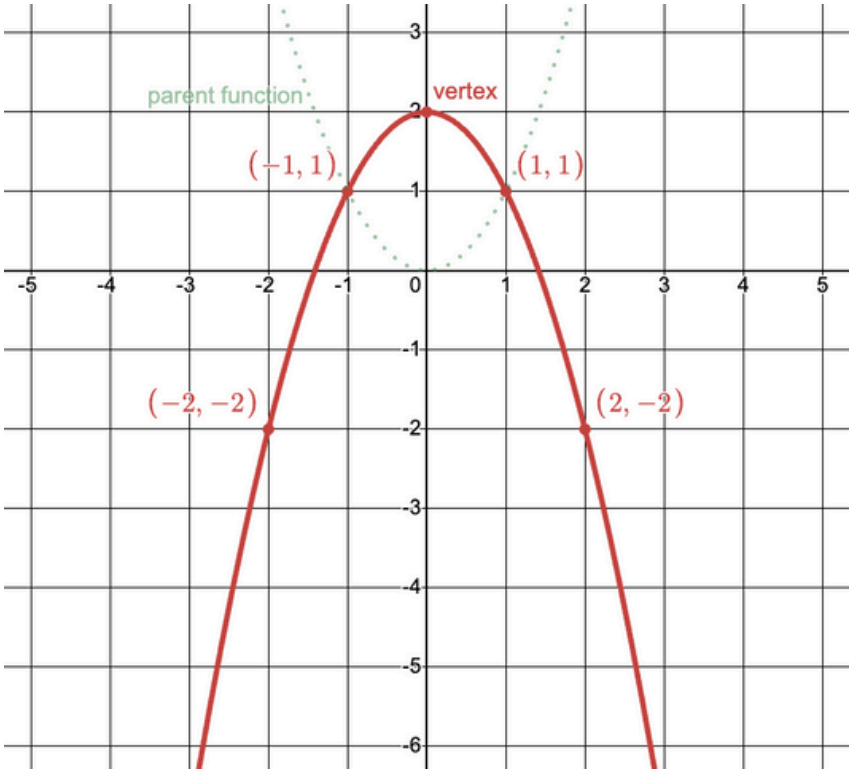
vertex :
(0, 2)

y – intercept :
when $x = 0$, $y = +2$

shape :
a = –1 so the shape doesn't change but it is flipped over the x – axis

Closest points :
over 1 *and* **down** 1

Next points :
over 2 *and* **down** 4



Example #3

$y = 2(x - 1)^2 + 1$

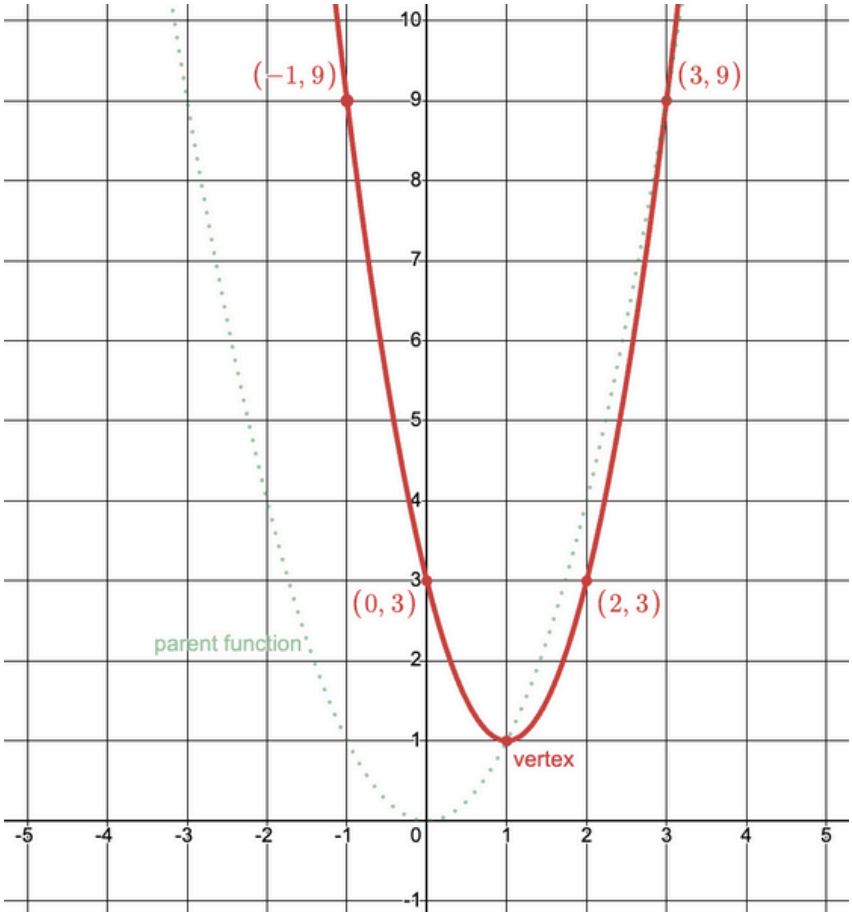
vertex :
(1, 1)

y – intercept :
when $x = 0$, $y = +3$

shape :
a = 2 so the y – values of the parent function get multiplied by 2

Closest points :
over 1 *and up* 2

Next points :
over 2 *and up* 8



Example #4

$y = 2x^2 - 4x + 4$

vertex form :
$y = 2(x-1)^2 + 2$

complete the square :

$\frac{y}{2} = x^2 - 2x + 2$

$\frac{y}{2} - 2 = x^2 - 2x$

$\frac{y}{2} - 2 + \left(\frac{-2}{2}\right)^2 = x^2 - 2x + \left(\frac{-2}{2}\right)^2$

$\frac{y}{2} - 2 + 1 = x^2 - 2x + (-1)^2$

$\frac{y}{2} - 1 = (x-1)^2$

$\frac{y}{2} = (x-1)^2 + 1$

$y = 2(x-1)^2 + 2$

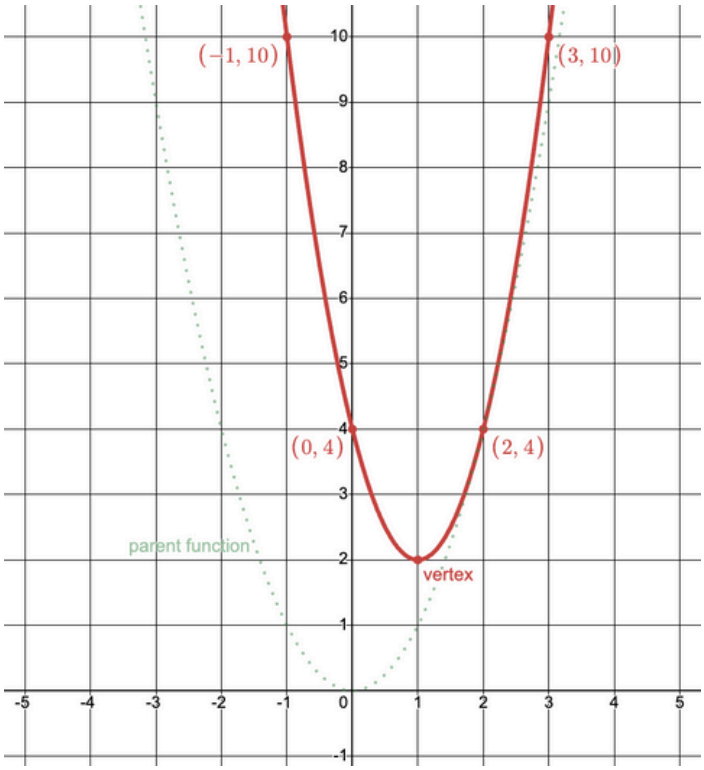
vertex :
(1, 2)

y – intercept :
when $x = 0$, $y = +4$

shape :
a = 2 so the y – values of the parent function get multiplied by 2

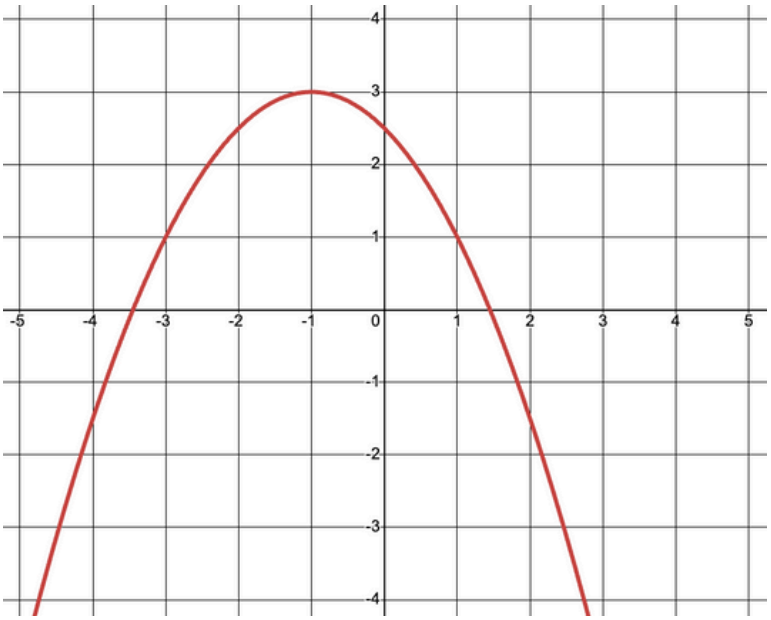
Closest points :
over 1 and up 2

Next points :
over 2 and up 8



Example #5

Find the equation from the graph:



Vertex Form :
$y = a(x-h)^2 + k$

vertex (from our graph) :
$(h, k) \rightarrow (-1, 3)$

$y = a(x-(-1))^2 + 3$

$y = a(x+1)^2 + 3$

to find **a** *:*
subtstitute another known point into the equation.

$(x, y) \rightarrow (1, 1)$

$1 = a(1+1)^2 + 3$
$1 = a(2)^2 + 3$
$-2 = 4a$
$a = -\frac{1}{2}$

so our equation in vertex form is :

$$y = -\frac{1}{2}(x+1)^2 + 3$$