Creating Quadratic Equations

Level 1 - Create an equation from a table of values when the y-intercept is given
Level 2 - Create an equation from a table of values when the y-intercept is not given
Level 3 - Create an equation from a table when the input values do not step up by 1

Math Ketchup

Quadratic equations model relationships where the rate of change is not constant. The standard from of a quadratic equation is given in the form:

$y = ax^2 + bx + c$

If we are given a table of values we can test for the quadratic relationship using the idea of **second differences. From here, we can find:**

- the **a** value using the second difference.
- the **c** value (y-intercept, when x = 0) and the **b** value using data from our table of values.

<u>Scenario 1:</u> The y-intercept is given and the x-values increase by 1

- When the x-values increase by 1 we can find our **a** value but using the rule: **2^{nd} difference = 2a**
- The **c** value is our y-intercept (when x = 0).

Example 1: Find the quadratic equation give the table of values given the y-intercept and the x values increase by 1.

x	-2	-1	0	1	2
y	4	0	-2	-2	0

first difference	-4	-2	0	2

second difference	2	2	2

We can see that we have a common second difference so we know it is quadratic.

- Using the rule, 2^{nd} difference = 2a shows that our **a = "1"**
- Our c value is the y-intercept (where x = 0), giving **c = "-2"**

From here we can substitute a pair of values from the table into our equation to find b.

$y = ax^2 + bx + c$
$y = x^2 + bx - 2$

$-2 = (1)^2 + b(1) - 2$

b = -1

a = 1
b = -1
c = -2

Final Equation: **$y = x^2 - x - 2$**

Scenario 2: The y-intercept is ***not given*** and the x-values increase by 1

- When the x-values increase by 1 we can find our **a** value but using the rule: **2nd difference = 2a**
- Use two points from our data table to create a system of 2 equations.
- Solve the system to find the values for **b** and **c**.

Example 1: Find the quadratic equation give the table of values.

x	1	2	3	4	5
y	0	-3	-4	-3	0

first difference	-3	-1	1	3

second difference	2	2	2

We can see that we have a common second difference so we know it is quadratic.

- Using the rule, 2nd difference = 2a shows that our **a = "1"**

We now know our equation will be in form $y = x^2 + bx + c$ because a = 1.

Since the table of values does not give the y-intercept we need to create a system of two equations using two different pairs of values from our table. Let's use the points (1, 0) and (5, 0).

$0 = (1)2 + b(1) + c$
and
$0 = (5)2 + b(5) + c$

$1 + b + c = 0$
$25 + 5b + c = 0$

Solving this system gives,
b = -6 and c = -7

Final Equation: $\mathbf{y = x^2 - 6x - 7}$

Scenario 3: The x-values do not increase by 1

- When the x-values ***do not*** increase by 1 we can find our **a** value but using the rule:
  2nd difference = 2·a·(step increase)²
- Then solve for **b** and **c** as we have done previously

step increase = 2

Example 1: Find the quadratic equation give the table of values.

x	-4	-2	0	2	4
y	29	7	1	11	37

first difference	-22	-6	10	26

second difference	16	16	16

2nd difference = $2 \cdot \mathbf{a} \cdot (\text{step increase})^2$ → $16 = 2 \cdot \mathbf{a} \cdot (2)^2$ → **a = 2**

Find **b** and **c** as we've done previously to get $\mathbf{y = 2x^2 + x + 1}$