Solving Equations by Completing the Square

Level 1 - Solve basic equations by Completing the Square

Level 2 - Solve equations involving fractions while Completing the Square

Level 3 - Solve by eliminating the algebraic fraction and then Completing the Square

Sometimes quadratic equations are not easy to factor. Completing the square is a reliable method that works for all quadratic equations. This method rewrites a quadratic expression as a perfect square, making it easier to solve for the variable.

Completing the Square Steps:

1. From general form $x^2 + bx + c = 0$, move the constant (c) to the other side of the equation.
2. Add $\left(\frac{b}{2}\right)^2$ to both sides. This creates a perfect square on one side.
3. Factor the perfect square and simplify the other side.
4. Solve for the variable.

Example #1	Example #2	Example #3
$x^2 + 6x + 5 = 0$	$x^2 - 3x - 2 = 0$	$\dfrac{x}{2} = \dfrac{-3x-7}{x+2}$

Example #1

step 1 → move the constant to the other side :

$$x^2 + 6x = -5$$

step 2 → add $\left(\frac{b}{2}\right)^2$ to both sides :

$$x^2 + 6x + \left(\frac{+6}{2}\right)^2 = -5 + \left(\frac{+6}{2}\right)^2$$

$$x^2 + 6x + (+3)^2 = -5 + (+3)^2$$

step 3 → factor the perfect square and simplify the other side :

$$(x + 3)^2 = 4$$

step 4 → solve for x :

$$x + 3 = \pm 2$$

$$\boxed{x = -1 \ or \ x = -5}$$

Example #2

step 1 → $x^2 - 3x = 2$

step 2 → $x^2 - 3x + \left(\frac{+3}{2}\right)^2 = 2 + \left(\frac{+3}{2}\right)^2$

$$x^2 - 3x + \left(\frac{+3}{2}\right)^2 = 2 + \frac{9}{4}$$

step 3 → $\left(x + \frac{3}{2}\right)^2 = \frac{8}{4} + \frac{9}{4}$

$$\left(x + \frac{3}{2}\right)^2 = \frac{17}{4}$$

step 4 → $x + \frac{3}{2} = \pm\sqrt{\frac{17}{4}}$

$$x + \frac{3}{2} = \frac{\pm\sqrt{17}}{2}$$

$$x = -\frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

$$\boxed{x = \frac{-3 \pm \sqrt{17}}{2}}$$

Example #3

clear the fraction, expand and move variables to one side :

$$x(x + 2) = 2(-3x - 7)$$

$$x^2 + 2x = -6x - 14$$

step 1 → $x^2 + 8x = -14$

step 2 → $x^2 + 8x + \left(\frac{+8}{2}\right)^2 = -14 + \left(\frac{+8}{2}\right)^2$

$$x^2 + 8x + (+4)^2 = -14 + (+4)^2$$

step 3 → $(x + 4)^2 = 2$

step 4 → $(x + 4) = \pm\sqrt{2}$

$$\boxed{x = -4 \pm \sqrt{2}}$$

Remember:

- The leading coefficient, **"a"**, must be **1**. If it's not you must divide all terms by **"a"** first.
- Always move the constant term to the other side before completing the square
- The shortcut to factoring the perfect square is highlighted!