

Solving Rational Equations (Quadratics)



Level 2 - If possible, solve rational equations with binomial denominators

Level 3 - If possible, solve rational equations after factoring

Rational equations are equations that contain fractions with variables in the denominator. Here, we are looking to solve rational equations that lead to quadratic equations.

The key is to clear the fractions first, then solve carefully and check for extraneous solutions (solutions that make a denominator equal to zero).

Key Skills:

- Multiply both sides by the least common denominator (LCD) to clear fractions.
- Solve the resulting equation, which is often a quadratic.
- Check all solutions in the original equation to make sure they are valid.

<p>Example #1</p> $\frac{6}{x+1} = \frac{x}{x-1}$	<p>Example #2</p> $\frac{x}{x-3} = \frac{10}{x^2-9}$	<p>Example #3</p> $x + \frac{6}{x-3} = \frac{2x}{x-3}$
<p>LCD: $(x+1)(x-1)$</p> <p>multiply both sides by the LCD:</p> $\frac{6(x+1)(x-1)}{\cancel{x+1}} = \frac{x(x+1)(x-1)}{\cancel{x-1}}$ $6(x-1) = x(x+1)$ $6x - 6 = x^2 + x$ $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = 2 \text{ or } x = 3$ </div> <p>check answers in original equation: → both solutions are valid.</p>	<p>factor where possible:</p> $\frac{x}{x-3} = \frac{10}{(x+3)(x-3)}$ <p>LCD: $(x+3)(x-3)$</p> <p>multiply both sides by the LCD:</p> $\frac{x(x+3)(\cancel{x-3})}{\cancel{x-3}} = \frac{10(x+3)(\cancel{x-3})}{(\cancel{x+3})(\cancel{x-3})}$ $x(x+3) = 10$ $x^2 + 3x = 10$ $x^2 + 3x - 10 = 0$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $(x+5)(x-2) = 0$ </div> $x = -5 \text{ or } x = 2$ <p>check answers in original equation: → both solutions are valid</p>	<p>LCD: $(x-3)$</p> <p>multiply each term by the LCD:</p> $x(x-3) + \frac{6(x-3)}{\cancel{x-3}} = \frac{2x(x-3)}{\cancel{x-3}}$ $x^2 - 3x + 6 = 2x$ $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2 \text{ or } x = 3$ <p>check answers in original equation: → $x = 2$ is allowed → $x = 3$ gives a zero in a denominator → 3 is not a valid solution</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = 2$ </div>

Remember:

- Start by factoring wherever possible
- Use the LCD to clear the fractions.
- Check your answers to avoid extraneous solutions.