

Creating Quadratic Equations



Level 1 - Create an equation from a table of values when the y-intercept is given

Level 2 - Create an equation from a table of values when the y-intercept is not given

Level 3 - Create an equation from a table when the input values do not step up by 1

Quadratic equations model relationships where the rate of change is not constant. The standard form of a quadratic equation is given in the form:

$$y = ax^2 + bx + c$$

If we are given a table of values we can test for the quadratic relationship using the idea of **second differences**. From here, we can find:

- the **a** value using the second difference.
- the **c** value (y-intercept, when $x = 0$) and the **b** value using data from our table of values.

Scenario 1: The y-intercept is given and the x-values increase by 1

- When the x-values increase by 1 we can find our **a** value but using the rule: **2nd difference = 2a**
- The **c** value is our y-intercept (when $x = 0$).

Example 1: Find the quadratic equation given the table of values given the y-intercept and the x values increase by 1.

x	-2	-1	0	1	2
y	4	0	-2	-2	0

first difference	-4	-2	0	2
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second difference	2	2	2
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We can see that we have a common second difference so we know it is quadratic.

- Using the rule, 2^{nd} difference = $2a$ shows that our **a = "1"**
- Our c value is the y-intercept (where $x = 0$), giving **c = "-2"**

From here we can substitute a pair of values from the table into our equation to find b.

$$\begin{array}{l}
 y = ax^2 + bx + c \\
 y = x^2 + bx - 2 \\
 -2 = (1)^2 + b(1) - 2 \\
 \mathbf{b = -1}
 \end{array}
 \left. \vphantom{\begin{array}{l} y = ax^2 + bx + c \\ y = x^2 + bx - 2 \\ -2 = (1)^2 + b(1) - 2 \\ \mathbf{b = -1} \end{array}} \right\}
 \begin{array}{l}
 \mathbf{a = 1} \\
 \mathbf{b = -1} \\
 \mathbf{c = -2}
 \end{array}
 \left. \vphantom{\begin{array}{l} \mathbf{a = 1} \\ \mathbf{b = -1} \\ \mathbf{c = -2} \end{array}} \right\}
 \text{Final Equation: } \mathbf{y = x^2 - x - 2}$$

Scenario 2: The y-intercept is not given and the x-values increase by 1

- When the x-values increase by 1 we can find our **a** value but using the rule: **2nd difference = 2a**
- Use two points from our data table to create a system of 2 equations.
- Solve the system to find the values for **b** and **c**.

Example 1: Find the quadratic equation give the table of values.

x	1	2	3	4	5
y	0	-3	-4	-3	0

first difference	-3	-1	1	3
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second difference	2	2	2
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We can see that we have a common second difference so we know it is quadratic.

- Using the rule, 2nd difference = 2a shows that our **a = "1"**

We now know our equation will be in form $y = x^2 + bx + c$ because $a = 1$.

Since the table of values does not give the y-intercept we need to create a system of two equations using two different pairs of values from our table. Let's use the points (1, 0) and (5, 0).

$$\begin{array}{l}
 0 = (1)^2 + b(1) + c \\
 \text{and} \\
 0 = (5)^2 + b(5) + c
 \end{array}
 \left. \begin{array}{l}
 1 + b + c = 0 \\
 25 + 5b + c = 0
 \end{array} \right\} \begin{array}{l}
 \text{Solving this system gives,} \\
 b = -6 \text{ and } c = +5
 \end{array} \left. \right\} \text{Final Equation: } y = x^2 - 6x + 5$$

Scenario 3: The x-values do not increase by 1

- When the x-values do not increase by 1 we can find our **a** value but using the rule: **2nd difference = 2·a·(step increase)²**
- Then solve for **b** and **c** as we have done previously

Example 1: Find the quadratic equation give the table of values.

x	-4	-2	0	2	4
y	29	7	1	11	37

first difference	-22	-6	10	26
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second difference	16	16	16
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↗ step increase = 2

2nd difference = 2·a·(step increase)² \longrightarrow $16 = 2 \cdot a \cdot (2)^2$ \longrightarrow **a = 2**

Find **b** and **c** as we've done previously to get **$y = 2x^2 + x + 1$**